Dimensions

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1 A gas bubble from an explosion under water oscillates with a time period T and proportional to $p^a d^b E^c$ where p is the static pressure, d is the density of water and E is the total energy of explosion. Find the value of a, b, c

sol
$$T \propto p^a d^b E^c$$

Equating dimensionally on both sides

$$[T] = [ML^{-1}T^{-2}]^a [ML^{-3}]^b [ML^2T^{-2}]^c$$

$$[M^0 L^0 T] = [M^{a+b+c}][L^{-a-3b+2c}][T^{-2a-2c}]$$

$\therefore$

$0 = a + b + c$ — (1)

$0 = -a - 3b + 2c$ — (11)

$1 = -2a - 2c$ — (111)

$1 \times (111) + 2(1)$, we get

$$2b = 1$$
$$b = 1/2$$

$(1) + (11)$

$$0 + 0 = -2b + 3c$$
$$3c = 2b$$
$$c = \frac{2}{3}\left(\frac{1}{2}\right)$$
$$= 1/3$$

From (1), $a = -b - c$

$$= -\frac{1}{2} - \frac{1}{3}$$
$$= \frac{-3-2}{6}$$
$$= -5/6$$

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2 Using dimension, convert 4 Joule to erg

Sol Dimension of work = $[ML^2T^{-2}]$

SI system

$M_1 = 1kg$

$L_1 = 1m$

$T_1 = 1s$

CGS system

$M_2 = 1g$

$L_2 = 1cm$

$T_2 = 1s$

$n_1 = 4$, $n_2 = ?$

$$n_1[M_1 L_1^2 T_1^{-2}] = n_2[M_2 L_2^2 T_2^{-2}]$$

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^1\left[\frac{L_1}{L_2}\right]^2\left[\frac{T_1}{T_2}\right]^{-2}$$

$$= 4\left[\frac{1kg}{1g}\right]^1\left[\frac{1m}{1cm}\right]^2\left[\frac{1s}{1s}\right]^{-2}$$

$$= 4\left[\frac{1000g}{1g}\right]^1\left[\frac{100cm}{1cm}\right]^2\left[\frac{1s}{1s}\right]^{-2}$$

$$= 4 \times 1000 \times 100 \times 100$$

$$\therefore n_2 = 4 \times 10^7$$

$$\therefore 4 \text{ Joule} = 4 \times 10^7 \text{ erg}$$

3 In CGS system, force is 100 dyne. In another system where the fundamental physical quantities are kilogram, meter, and minute, find the magnitude of force

Sol Dimension of force = $[MLT^{-2}]$

$n_1 = 100$, $n_2 = ?$

cgs	New system
$M_1 = 1g$	$M_2 = 1kg$
$L_1 = 1cm$	$L_2 = 1m$
$T_1 = 1s$	$T_2 = 1min$

$$n_1[M_1L_1T_1^{-2}] = n_2[M_2L_2T_2^{-2}]$$

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^1\left[\frac{L_1}{L_2}\right]\left[\frac{T_1}{T_2}\right]^{-2}$$

$$= 100\left[\frac{1g}{1kg}\right]\left[\frac{1cm}{1m}\right]\left[\frac{1s}{1min}\right]$$

$$= 100\left[\frac{1g}{1000g}\right]\left[\frac{1cm}{100cm}\right]\left[\frac{1s}{60s}\right]^{-2}$$

$$= 100 \times \frac{1}{1000} \times \frac{1}{100} \times 60 \times 60$$

$$n_2 = 3.6$$

4 A calorie is a unit of heat or energy and it equal about 4.2 J, where $1J = 1kg\,m^2s^{-2}$. suppose we employ a system of units in which unit of mass is α kg, the unit of length is β m and the unit of time is γ s. show that a calorie has a magnitude of $4.2\,\alpha^{-1}\beta^{-1}\gamma^2$ in terms of new unit

Sol Dimension of energy = $[ML^2T^{-2}]$

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SI system | New system

$M_1 = 1 kg$ $\qquad$ $M_2 = \alpha kg$

$L_1 = 1 m$ $\qquad$ $L_2 = \beta m$

$T_1 = 1 s$ $\qquad$ $T_2 = \gamma s$

$n_1 = 4.2$ (why?)

$n_2 = ?$

$$n_1 [M_1 L_1^2 T_1^{-2}] = n_2 [M_2 L_2^2 T_2^{-2}]$$

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]\left[\frac{L_1}{L_2}\right]^2 \left[\frac{T_1}{T_2}\right]^{-2}$$

$$= 4.2 \left[\frac{1 kg}{\alpha kg}\right]\left[\frac{1 m}{\beta m}\right]^2 \left[\frac{1 s}{\gamma s}\right]^{-2}$$

$$= 4.2 \left[\frac{1}{\alpha}\right]\left[\frac{1}{\beta}\right]^2 \left[\frac{1}{\gamma}\right]^{-2}$$

$$= 4.2 \alpha^{-1} \beta^{-2} \gamma^{2}$$

5 If the velocity of light (c), gravitational constant (G) and Planck's constant (h) are chosen as fundamental units, find the dimension of mass in new system

Sol $\quad m \propto c^a G^b h^z$

$$m = k c^a G^b h^z$$

$$[M L^0 T^0] = [LT^{-1}]^a [M^{-1} L^3 T^{-2}]^b [M L^2 T^{-1}]^z$$

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$$[M L^0 T^0] = [M^{-b+z}][L^{a+3b+2z}][T^{-a-2b}$$

$\therefore \quad 1 = -b + z \quad - (1)$

$0 = a + 3b + 2z \quad - (II)$

$0 = -a - 2b - z \quad - (III)$

(II) + (III)

$0 = b + z \quad - (iv)$

(iv) + (1)

$1 = 2z$

$\therefore \quad z = 1/2$

$\therefore \quad b = -1/2$

Now $\quad a = -2b - z$

$= -2\left(-\frac{1}{2}\right) - \left(\frac{1}{2}\right)$

$= 1 - \frac{1}{2}$

$= \frac{1}{2}$

$\therefore \quad a = 1/2, \quad b = -\frac{1}{2}, \quad z = 1/2$

Hence $\quad m = K c^{1/2} G^{-1/2} h^{1/2}$

or $\quad m \propto \sqrt{\frac{ch}{G}}$

Home Task

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1) The position of a particle at time t is given by the relation $x(t) = \left(\frac{v_0}{\alpha}\right)(1 - e^{-\alpha t})$, where v_0 is a constant and $\alpha > 0$. Find the dimension of v_0 and α

2 Convert 1MW power on a new system having basic units of mass, length and time as 10kg, 1dm and 1min respectively

3 If velocity (V), force (F) and energy (E) are taken as fundamental units, then find the dimensional formula for mass

4 The potential energy of a particle varies with distance x from a fixed origin as $U = \frac{A\sqrt{x}}{x^2 + B}$, where A and B are dimensional constants. Find the dimensional formula of AB

5 The equation of a wave is given by $Y = A \sin \omega\left(\frac{x}{V} - k\right)$, where ω is the angular velocity and V is the linear velocity. Find the dimension of K