Ampere's Circuital Law:

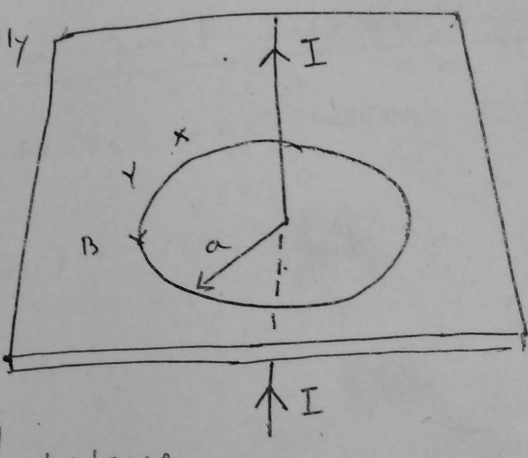
Ampere's circuital law states that the line integral of the magnetic field around any closed path in free space is equal to absolute permeability (μ_0) times the net current ~~cur~~ passing through any surface enclosed by the closed path

Mathematically $\oint \vec{B} \cdot \vec{dl} = \mu_0 I$, where

$\vec{B}$ is the magnetic field,

$\vec{dl}$ is the small element

Proof: consider an infinitely long straight conductor carrying current I. The magnetic field lines are produced around the conductor as concentric circles



The magnetic field due to this current carrying infinite conductor at a distance a is given by using Biot Savart Law

$$B = \frac{\mu_0}{4\pi}\left(\frac{2I}{a}\right)$$

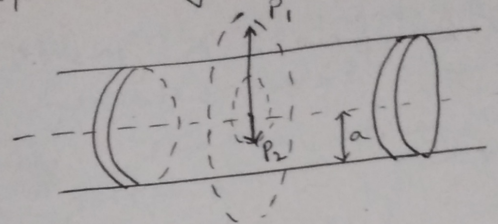
consider a circle of radius a around a wire (amperian loop).

Let XY be a small element of length dl

$\vec{dl}$ and $\vec{B}$ are in the same direction.

$\vec{B} \cdot \vec{dl} =$

Magnetic field due to current carrying circular wire of infinite length using Ampere's law



Consider a portion of a circular wire of infinite length. Let a be the radius of the wire. and steady current I be flowing through it. The flow of current in the wire gives rise to magnetic field. The magnetic field lines are concentric circles with their centres on the axis of the wire.

I Magnetic field intensity at a point outside the wire

Let P_1 be the point outside the wire such that $r_1 > a$

According to Ampere's circuital law:

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$$

$$\oint B\,dl \cos 0 = \mu_0 I$$

$$B \times 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

II Magnetic field intensity on the surface of a wire:

Here $r = a$

$$\therefore B = \frac{\mu_0 I}{2\pi a}$$

III Magnetic field intensity at a point inside the wire.

Consider a point P_2 inside the wire

Here $r < a$

If the current flows only along the surface of the wire, then

$$\oint \vec{B} \cdot \vec{dl} = 0 \qquad (\because I = 0, \text{ inside the circle of radius } r < a)$$

$\therefore B = 0$

If the current is uniformly distributed throughout the cross-section of the wire, then the current through the circle of radius $r < a$ is

I' = current per unit area of the wire × area of the circle of radius r

$$= \frac{I}{\pi a^2} \times \pi r^2$$

$$= \frac{I r^2}{a^2}$$

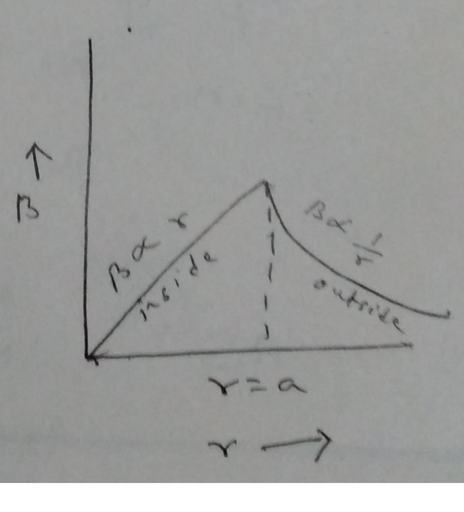
According to Ampere's circuital law

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I'$$

$$B \times 2\pi r = \mu_0 \frac{I r^2}{a^2}$$

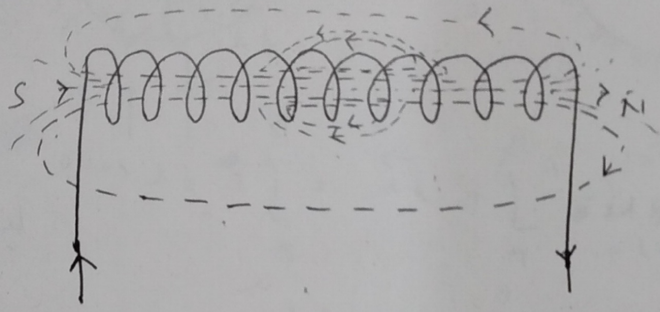
$$B = \frac{\mu_0 I r}{2\pi a^2}$$

Thus $B \propto r$



Straight Solenoid and Toroid

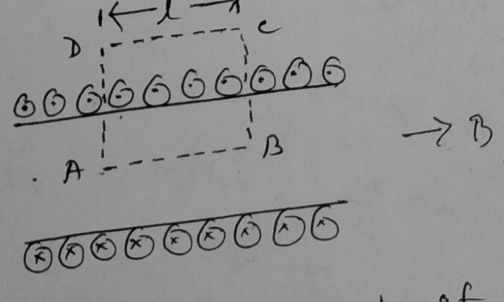
A cylindrical coil of many tightly wounds turns of insulated wire with diameter of the coil smaller than its length is called a solenoid



Consider a very long solenoid having n turns per unit length of solenoid.

The magnetic field outside a very long solenoid is very weak and can be neglected. The magnetic field inside the solenoid is almost uniform, strong and directed along the axis of the solenoid.

Step 1 Let P be a point well inside a solenoid. Consider any rectangular loop ABCD (known as Amperian loop) passing through P



$\oint \vec{B} \cdot d\vec{l}$ = Line integral of magnetic field across loop ABCD

$$= \int_A^B \vec{B} \cdot d\vec{l} + \int_B^C \vec{B} \cdot d\vec{l} + \int_C^D \vec{B} \cdot d\vec{l} + \int_D^A \vec{B} \cdot d\vec{l}$$

$\vec{B}$ is $\perp$ to paths BC and AD

$$\therefore \int_B^C \vec{B}\cdot d\vec{l} = \int_D^A \vec{B}\cdot d\vec{l} = 0$$

Since CD is outside the solenoid, so $\vec{B}$ is taken as zero, $\int_C^D \vec{B}\cdot d\vec{l} = 0$.

(For path AB, direction of $d\vec{l}$ and $\vec{B}$ is same i.e. $\theta = 0$)

$$\therefore \oint \vec{B}\cdot d\vec{l} = \int_A^B \vec{B}\cdot d\vec{l} = \int_A^B B\, dl \cos 0$$

$$= B\int_A^B dl = Bl$$

<u>Step 2</u> According to Ampere's circuital law

$$\oint_{ABCD} \vec{B}\cdot d\vec{l} = \mu_0 \times \text{net current enclosed by loop.}$$

$$Bl = \mu_0 \times \text{no of turns in loop ABCD} \times I$$

$$Bl = \mu_0 n l I$$

$$B = \mu_0 n I$$

Thus, magnetic field well within an infinitely long solenoid is

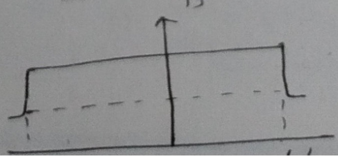
$$B = \mu_0 n I$$

Representing $n = \frac{N}{l}$, where

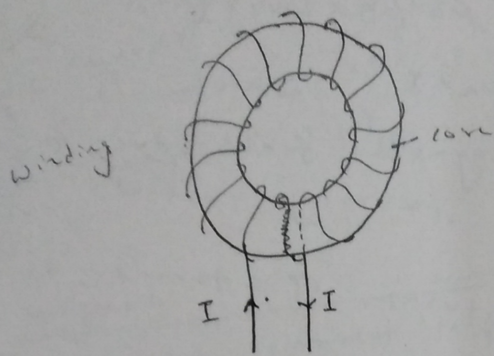
N = No of turn of solenoid

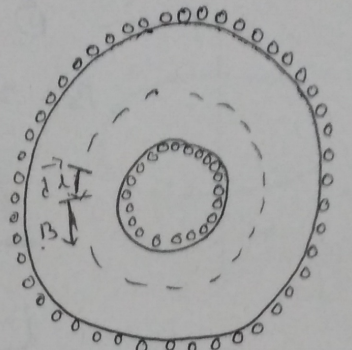
$$B = \frac{\mu_0 N I}{l}$$

[N.B Magnetic field at each end of a long solenoid reduces to half of the value given by $B_{ends} = \frac{1}{2}\mu_0 n I$]



Magnetic Field due to a Toroid carrying current





(A toroid can be considered as a ring shaped closed solenoid)

Consider a toroid having n turns per unit ~~volume~~ length. The magnetic lines of force mainly remain in the core of toroid and are in the form of concentric circles.

Consider such a circle of radius r

$$\oint \vec{B}\cdot\vec{dl} = \oint B\,dl\cos\theta$$

By symmetry, magnetic field B in the coil is constant and is along the tangent to path $\vec{dl}$

$$\therefore \oint \vec{B}\cdot\vec{dl} = \oint B\,dl\cos 0$$

$$= B\oint dl \quad \text{—(1)}$$

$$= B\times 2\pi r$$

From Ampere's circuital law

$\oint \vec{B}\cdot\vec{dl} = \mu_0 \times$ net current enclosed by the circle of radius r

$= \mu_0 \times$ total no of turns $\times I$

$= \mu_0 (n \times 2\pi r) I$ —(2)

From 1 and 2

$$B \times 2\pi r = \mu_0 (n \times 2\pi r) I$$

$B = \mu_0 n I$, which is the ~~total~~ magnetic field due to a toroid carrying current.

if N = Total no. of turns of toroid then,

$$N = n \times 2\pi r$$

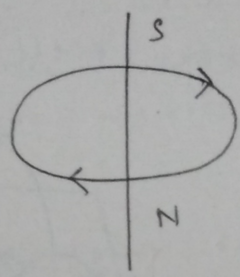
or $$n = \frac{N}{2\pi r}$$

$$\therefore \quad B = \frac{\mu_0 N I}{2\pi r}$$

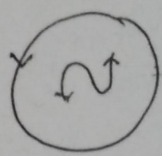
(For any point inside the empty space surrounded by a toroid and outside the toroid, magnetic field B is zero because the net current enclosed in these space is zero. But magnetic field is not exactly zero)

Current Loop as Magnetic Dipole.

· current loop behaves as a magnet i.e. magnetic dipole.



<u>Magnetic dipole moment (M) of the current loop</u>



From back current appears clockwise and south pole appears.

Thus, upper face behaves as N-pole and lower face as S-pole

Torque acting on a current carrying loop placed in the magnetic field is

$\tau = IAB \sin\theta$

I = current in the loop
A = Area of loop
θ = angle betn $\vec{B}$ and normal to the plane of loop

<u>IA</u> is known as <u>magnetic dipole moment</u> of the current loop.
It is denoted by m

$\therefore$ m = IA

If the loop has n turns, then

m = nIA

In vector form,

$\vec{m} = nI\vec{A}$

S.I. unit is ampere-m² (i.e. $A\text{-}m^2$) or $\frac{\text{Joule}}{\text{Tesla}}$

Dimensional formula = $M^0 L^2 T^0 A$