Cl-XII (Maths).

Increasing and decreasing Function.

Ex-1 If $x>0$, prove that $x > \log(1+x) > \frac{x}{1+x}$.

Soln $f(x) = x - \log(1+x)$

$\therefore f(0) = 0 \qquad f'(x) = 1 - \frac{1}{1+x} = \frac{x}{1+x}$.

If $x>0$ then $f'(x) > 0$

$\therefore f(x)$ is increasing function for $x>0$.

$\therefore f(x) > f(0) = 0 \quad \forall x>0$

$\therefore x > \log(1+x) \quad \text{---(1)}$

Again $g(x) = \log(1+x) - \frac{x}{1+x}$.

$\therefore g'(x) = \frac{1}{1+x} - \frac{(1+x)-x}{(1+x)^2} = \frac{x}{(1+x)^2}$.

$\therefore g'(x) > 0$ for $x>0$

$\therefore g(x)$ is increasing function $\forall x>0$

$\therefore g(x) > g(0) = 0 \quad \forall x>0$

$\therefore \log(1+x) > \frac{x}{1+x} \quad \text{---(11)}$.

from (1) and (11) we get $x > \log(1+x) > \frac{x}{1+x}$.

Ex-2 Find the values of parameter a for which the function $y = ax^3 + 3x^2 + (2a+1)x + 1000$ is strictly decreasing for all real values of x.

Soln. $y = ax^3 + 3x^2 + (2a+1)x + 1000$

$\therefore \frac{dy}{dx} = 3ax^2 + 6x + 2a + 1$

$= 3a\left(x + \frac{1}{a}\right)^2 + 2a + 1 - \frac{3}{a}$.

Now y is strictly decreasing function.

$\therefore \frac{dy}{dx} < 0 \quad \therefore 3a < 0 \quad \therefore a < 0$

and $2a + 1 - \frac{3}{a} < 0$ or $2a + 3 - 2 - \frac{3}{a} < 0$

or $(2a+3) - \frac{1}{a}(2a+3) < 0$

or $(2a+3)\left(1 - \frac{1}{a}\right) < 0$

or $(2a+3)(a-1) < 0$

$\Rightarrow a < -3/2 \quad [\because a < 0]$

Cl - XII (Maths).

S.H-3 Show that for all real values of θ the function $\frac{2\sin\theta+\cos\theta}{3\sin\theta+4\cos\theta}$ is increasing.

Soln. $y = \frac{2\sin\theta+\cos\theta}{3\sin\theta+4\cos\theta}$

$\therefore \frac{dy}{d\theta} = \frac{(3\sin\theta+4\cos\theta)(2\cos\theta-\sin\theta)-(2\sin\theta+\cos\theta)(3\cos\theta-4\sin\theta)}{(3\sin\theta+4\cos\theta)^2}$

$= \frac{8\cos^2\theta-3\sin^2\theta+2\sin\theta\cos\theta-(3\cos^2\theta-8\sin^2\theta+2\sin\theta\cos\theta)}{(3\sin\theta+4\cos\theta)^2}$

$= \frac{5\cos^2\theta+5\sin^2\theta}{(3\sin\theta+4\cos\theta)^2} = \frac{5}{(3\sin\theta+4\cos\theta)^2} > 0$

$\therefore$ The required function is always increasing function.

SH-4. Find the intervals in which the function $f(x) = \frac{3}{x}+\frac{x}{3}$ is (i) increasing (ii) decreasing.

Soln. $\because f(x) = \frac{3}{x}+\frac{x}{3}$ $\quad f'(x) = -\frac{3}{x^2}+\frac{1}{3}, x \neq 0$

(i) when $f(x)$ is increasing, $f'(x) > 0$

$\Rightarrow -\frac{3}{x^2}+\frac{1}{3} > 0$ or $-\frac{3}{x^2} > -\frac{1}{3}$

or $9 < x^2$

$\Rightarrow 3 < x$ or $x < -3$

$\therefore f(x)$ is increasing in $(-\infty,-3) \cup (3,\infty)$.

(ii) when $f(x)$ is decreasing, $f'(x) < 0$ or $-\frac{3}{x^2}+\frac{1}{3} < 0$

$\Rightarrow x^2 < 9$

$\Rightarrow -3 < x < 3$ and $x \neq 0$

[$\because f(x)$ is not defined at $x=0$]

The required interval $= (-3,0) \cup (0,3)$

SN-5. Show that the function $f(x) = x^2 - x + 1$ is neither increasing nor decreasing in $(-1, 1)$.

Solⁿ. $f(x) = x^2 - x + 1$

$\therefore f'(x) = 2x - 1$ $\quad$ Now in

$\therefore -1 < x < \frac{1}{2}$, $f'(x) < 0$

and $\frac{1}{2} < x < 1$, $f'(x) > 0$

$\therefore f(x)$ is neither increasing nor decreasing.

SN-6 Prove that the function $f(x) = x^3 - 6x^2 + 12x - 18$ is increasing for all $x \in R$.

Solⁿ $f(x) = x^3 - 6x^2 + 12x - 18$

$\therefore f'(x) = 3x^2 - 12x + 12$

$\Rightarrow f'(x) = 3(x-2)^2 \geq 0$ $\quad \therefore f(x)$ is increasing $\forall x \in R$.

S.N-7. Prove that the function $f(x) = 4x^3 + 6x^2 - 24x + 1$ decreases in the interval $(-2, 1)$.

Solⁿ. $f(x) = 4x^3 + 6x^2 - 24x + 1$

$\therefore f'(x) = 12x^2 + 12x - 24 = 12(x^2 + x - 2)$

$= 12(x+2)(x-1)$.

$\therefore$ Now in $-2 < x < 1$, $0 < x+2 < 3$ and $-3 < x-1 < 0$

$\Rightarrow 12(x+2)(x-1) < 0 \Rightarrow f'(x) < 0$

$\therefore f(x)$ decreasing in $(-2, 1)$.